

Comments on the role of field redefinition on renormalisation of $N = \frac{1}{2}$ supersymmetric pure gauge theory

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Abstract

We study one loop corrections to $N = \frac{1}{2}$ supersymmetric $SU(N) \times U(1)$ pure gauge theory. We calculate divergent contributions of the 1PI graphs contain the non-anti-commutative parameter C up to one loop corrections. We find the disagreement between component formalism and superspace formalism is because of the field redefinition in component case. We modify gaugino field redefinition and lagrangian. We show extra terms of lagrangian have been generated by λ redefinition and are necessary for the renormalisation of the theory. Finally we prove $N = \frac{1}{2}$ supersymmetric gauge theory is renormalisable up one loop corrections using standard method of renormalisation

1 Introduction

Theories defined on non-anti-commutative superspace have been studied extensively during last ten years [1, 2]. Superspace in such non-anti-commutative theories is a superspace whose fermionic supercoordinates are not anticommutative. One could construct a field theory in non-anti-commutative superspace in terms of superfields with the star-product where lagrangian is deformed from the original theory by the non-anti-commutative parameters.

Recently some renormalisability aspects of the non-anti-commutative field theories have been studied. It has been shown non-anti-commutative field theories are not power-counting renormalisable; however it has been discussed that they could be renormalisable if some additional terms have been added to the lagrangian in order to divergences to all orders [3]-[8]. The renormalisability of non-anti-commutative versions of the Wess-Zumino model has been discussed [3, 4], with explicit computations up to two loops [5]. The renormalisability of non-anti-commutative gauge theory with $N = \frac{1}{2}$ supersymmetry has been studied in [6, 8]. The authors in [6] show that the theory is renormalisable to all order of perturbation theory. The conditions of the renormalisability of non-anticommutative (NAC) field theories have been studied with explicit computations up one and two loops [9]-[15].

The renormalisability of supersymmetric gauge field theories has been discussed in WZ gauge [6, 7]. The explicit one loop corrections in component formalism have been done in [9]-[11]. The authors in [10, 11] have claimed the precise form of the lagrangian is not preserved by renormalisation. They have shown by explicit calculation that there are problems with assumption of gauge invariance which is required to rule out some classes of divergent structure in non-anti-commutative theory. From their calculation, one can see even at one loop divergent non-gauge-invariant terms are generated. In order to remove the non-gauge-invariant terms

and restore gauge invariance at one loops they introduce a one loop divergent field redefinition in the case of pure $N = \frac{1}{2}$ supersymmetry (i.e. no chiral matter).

On the other hand, the authors in [12, 13] have started from superspace formalism and discussed renormalisability and supergauge invariance. They proved that the field redefinition is not necessary and the original effective action is not only gauge but also supergauge invariant up one loop corrections. The disagreement between two approaches put a big question mark which approaches we should rely on in $N = \frac{1}{2}$ supersymmetric gauge theory.

In this paper we investigate the renormalisability of $N = \frac{1}{2}$ supersymmetric pure gauge theory at one-loop perturbative corrections in component formalism. We shall show $N = \frac{1}{2}$ supersymmetric gauge theory is renormalisable in a usual manner without any needs for field redefinition (there is not theoretical justification or interpretation for the field redefinition as mentioned by authors [10]) which leads to the lagrangian change. Therefore we shall prove two approaches lead to the same conclusion.

The paper is organized as follows: First we briefly review NAC supersymmetric gauge theories and their lagrangian. Then an explicit one-loop calculation of the three and four-point functions of the theory in the C-deformed sector is carried out to calculate the corrections. We show some anomaly terms appears in the 1PI functions which spoil the renormalisability of theory. Next we introduce extra terms to the original lagrangian in order to renormalise NAC pure gauge supersymmetric theory and calculate corrections which come from these new terms. Finally we discuss the source of the extra lagrangian, and show that these new terms have hidden because of the component λ redefinition [1, 20]), so in order to reproduce them one should reverse gaugino field redefinition.

2 The pure gauge supersymmetric action of NAC gauge theory

The original non-anticommutative theory defined in superfields appears to require a $U(N)$ gauge group. Here, at first we would like to consider $U(N)$ gauge theory for non-(anti)commutative (NAC) superspace. The action for an $N = 1/2$ supersymmetric $U(N)$ pure gauge theory is given by:

$$S = \int d^4x \left[Tr \left\{ -\frac{1}{2} F^{\mu\nu} F_{\mu\nu} - 2i\bar{\lambda}\bar{\sigma}^\mu (D_\mu \lambda) + D^2 \right\} - 2igC^{\mu\nu} Tr \{ F_{\mu\nu} \bar{\lambda}\bar{\lambda} \} + g^2 |C|^2 Tr \{ (\bar{\lambda}\bar{\lambda})^2 \} \right], \quad (1)$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu], \\ D_\mu \lambda &= \partial_\mu \lambda + ig[A_\mu, \lambda], \end{aligned} \quad (2)$$

and

$$A_\mu = A_\mu^A R^A, \quad \lambda = \lambda^A R^A, \quad D = D^A R^A, \quad (3)$$

Corresponding to any index a for $SU(N)$ we introduce the index $A = (0, a)$, so that A runs from 0 to $N^2 - 1$. with R^A being the group matrices for $U(N)$ in the fundamental representation. These satisfy

$$[R^A, R^B] = if^{ABC} R^C, \quad \{R^A, R^B\} = d^{ABC} R^C, \quad (4)$$

where f^{ABC} is completely antisymmetric, f^{abc} is the same as for $SU(N)$ and $f^{0bc} = 0$, while d^{ABC} is completely symmetric; d^{abc} is the same as for $SU(N)$, $d^{0bc} = \sqrt{2/N} \delta^{bc}$, $d^{00c} = 0$ and $d^{000} = \sqrt{2/N}$. In particular, $R^0 = \sqrt{\frac{1}{2N}} 1$. We have also

$$Tr \{ R^A R^B \} = \frac{1}{2} \delta^{AB} \quad (5)$$

The following identities hold in U(N) group and will be extensively used below

$$f^{ABL}f^{LCD} + f^{ACL}f^{LDB} + f^{ADL}f^{LBC} = 0, \quad (6)$$

$$f^{ABL}d^{LCD} + f^{ACL}d^{LDB} + f^{ADL}d^{LBC} = 0, \quad (7)$$

$$f^{ADL}f^{LBC} = d^{ABL}d^{LCD} - d^{ACL}d^{LDB}, \quad (8)$$

$$f^{IAJ}f^{JBK}f^{KCI} = -\frac{N}{2}f^{ABC}, \quad (9)$$

$$d^{IAJ}f^{JBK}f^{KCI} = -\frac{N}{2}d^{ABC}d^A c^B c^C. \quad (10)$$

Where $d^A = 1 + \delta^{0A}$, $c^A = 1 - \delta^{0A}$.

Upon substituting the above relations in eq. (1), we obtain the action in the U(N) case in the form:

$$S = \int d^4x \left[-\frac{1}{4}F^{\mu\nu A}F_{\mu\nu}^A - i\bar{\lambda}^A\bar{\sigma}^\mu(D_\mu\lambda)^A + \frac{1}{2}D^A D^A \right. \\ \left. - \frac{1}{2}igd^{ABC}C^{\mu\nu}F_{\mu\nu}^A\bar{\lambda}^B\bar{\lambda}^C + \frac{1}{8}g^2d^{ABE}d^{CDE}|C|^2(\bar{\lambda}^A\bar{\lambda}^B)(\bar{\lambda}^C\bar{\lambda}^D) \right] \quad (11)$$

With gauge coupling g , gauge field A_μ and gaugino λ .

Beside, definition for $F_{\mu\nu}$ and $D_\mu\lambda^a$ are given by:

$$F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A - gf^{ABC}A_\mu^B A_\nu^C, \\ D_\mu\lambda^A = \partial_\mu\lambda^A - gf^{ABC}A_\mu^B\lambda^C, \quad (12)$$

$C^{\mu\nu}$ is related to the non-anti-commutativity parameter $C^{\alpha\beta}$ by:

$$C^{\mu\nu} = C^{\alpha\beta}\epsilon_{\beta\gamma}\sigma_\alpha^{\mu\nu\gamma} \quad (13)$$

also, we have:

$$C^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\gamma}\sigma_\gamma^{\mu\nu\beta}C_{\mu\nu}, \quad (14)$$

where

$$\sigma_\alpha^{\mu\nu\beta} = \frac{1}{4}(\sigma^\mu_{\alpha\dot{\rho}}\bar{\sigma}^{\nu\dot{\rho}\beta} - \sigma^\nu_{\alpha\dot{\rho}}\bar{\sigma}^{\mu\dot{\rho}\beta}), \quad (15)$$

$$\bar{\sigma}_{\dot{\beta}}^{\mu\nu\alpha} = \frac{1}{4}(\bar{\sigma}^{\mu\dot{\alpha}\rho}\sigma^\nu_{\rho\dot{\beta}} - \bar{\sigma}^{\nu\dot{\alpha}\rho}\sigma^\mu_{\rho\dot{\beta}}). \quad (16)$$

The useful identity is:

$$|C|^2 = C^{\mu\nu}C_{\mu\nu}. \quad (17)$$

There are some properties of C in App A. In above Eqs., $C^{\alpha\beta}$ is the non-anti-commutativity parameter, and our conventions are consistent with ref [1]. The action for pure $N = 1/2$ supersymmetric gauge theory (Eq. 11) is invariant under the standard $U(N)$ gauge transformations and the $N = 1/2$ supersymmetry transformations. The standard $U(N)$ version of the NAC gauge theory is not renormalisable [1]. Therefore; we would like to present a $SU(N) \times U(1)$ lagrangian which has $N = \frac{1}{2}$ supersymmetric properties, so we introduce the following action:

$$S = \int d^4x \left[-\frac{1}{4}F^{\mu\nu A}F_{\mu\nu}^A - i\bar{\lambda}^A\bar{\sigma}^\mu(D_\mu\lambda)^A + \frac{1}{2}D^A D^A - \frac{1}{2}i\gamma^{ABC}d^{ABC}C^{\mu\nu}F_{\mu\nu}^A\bar{\lambda}^B\bar{\lambda}^C + \frac{1}{8}\gamma^{ABCDE}d^{ABE}d^{CDE}|C|^2(\bar{\lambda}^A\bar{\lambda}^B)(\bar{\lambda}^C\bar{\lambda}^D) \right], \quad (18)$$

One beauty of the above equation is one could easily switch between the original $U(N)$ theory and $SU(N) \times U(1)$ theory. In our work we define $\gamma^{abcde} = \gamma_0$, $\gamma^{abcd0} = \gamma_1$, $\gamma^{0b0de} = \gamma^{a0c0e} = \gamma^{0bc0e} = \gamma^{a00de} = \gamma_2$. Indeed we give them in terms of g and g_0 . They are given by:

$$\gamma^{abc} = g, \gamma^{ab0} = \gamma^{a0b} = g_0, \gamma^{0ab} = \frac{g^2}{g_0} \quad (19)$$

$$\gamma_0 = g^2, \gamma_1 = \left(\frac{g^2}{g_0}\right)^2, \gamma_2 = g_0^2 h \quad (20)$$

Where $h = 1$. The above action is similar to the $SU(N) \times U(1)$ action in ref [11].

The $N=1/2$ supersymmetry transformation is:

$$\delta A_\mu^A = -i\bar{\lambda}^A\bar{\sigma}_\mu\epsilon, \quad (21)$$

$$\delta\lambda_\alpha^A = i\epsilon_\alpha D^A + (\sigma^{\mu\nu}\epsilon)_\alpha[F_{\mu\nu}^A + \frac{1}{2}iC_{\mu\nu}\gamma^{ABC}d^{ABC}\bar{\lambda}^B\bar{\lambda}^C], \delta\bar{\lambda}_\alpha^A = 0, \quad (22)$$

$$\delta D^A = -\epsilon\sigma^\mu D_\mu\bar{\lambda}^A. \quad (23)$$

3 One-loop corrections

In our calculation, we use standard gauge fixing term

$$S_{gf} = \frac{1}{2\alpha} \int d^4x (\partial \cdot A)^2 \quad (24)$$

and consider the Feynman rules in the super-Feynman gauge ($\alpha = 1$).

In this section we first review the one-loop perturbative corrections to the undeformed $N = 1$ part of the theory. It has been shown that the quantum corrections of $N = 1$ part of the theory are not affected by C -deformation [9, 10]. Therefore; gauge field and gaugino anomalous dimensions and gauge β -functions are the same as those in the ordinary $N = 1$ case. The C -independent part of the bare action can be written as:

$$S_{C=0} = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu}^A - i \bar{\lambda}^A \bar{\sigma}^\mu \partial_\mu \lambda^A + i g f^{ABC} \bar{\lambda}^A \bar{\sigma}^\mu A_\mu^B \lambda^C + \frac{1}{2} D^A D^A \right] \quad (25)$$

The C -independent part of the action is renormalisable if one introduce bare fields and couplings according to:

$$A_{B\mu} = Z_A^{\frac{1}{2}} A_\mu, \quad \lambda_B = Z_\lambda^{\frac{1}{2}} \lambda, \quad g_B = Z_g g, \quad (26)$$

that Z_A, Z_λ and Z_g are known as a renormalisation constants. Also one can define:

$$\delta_1 = Z_A - 1, \quad \delta_2 = Z_\lambda - 1, \quad \delta_3 = Z_g Z_A^{\frac{1}{2}} Z_\lambda - 1, \quad (27)$$

finally, one should add the following counter terms to the lagrangian of theory in order to renormalise theory:

$$L_{counter-terms} = -\frac{1}{4} \delta_1 F^{\mu\nu} F_{\mu\nu}^A - i \delta_2 \bar{\lambda}^A \bar{\sigma}^\mu \partial_\mu \lambda^A + \delta_3 i g f^{ABC} \bar{\lambda}^A \bar{\sigma}^\mu A_\mu^B \lambda^C, \quad (28)$$

where,

$$Z_A = 1 + 2NL, \quad Z_\lambda = 1 - 2NL, \quad Z_g = 1 - 3NL, \quad (29)$$

and L is given by:

$$L = \frac{g^2}{16\pi^2\varepsilon}. \quad (30)$$

Here $\varepsilon = 4 - D$ is the regulator.

(We have given here the renormalisation constants corresponding to the $SU(N)$ sector of the $U(N)$ theory; those for the $U(1)$ sector, namely $Z_{\lambda^0}, Z_{A^0}, Z_{g_0}$ are given by omitting the terms in N and replacing g by g_0 .)

3.1 One-loop C deformed Corrections

In this part we will present on-loop graphs contributing to the new terms arising from C -deformed part of the action. The one-loop one-particle-irreducible (1PI) graphs of the C -deform $A\bar{\lambda}\bar{\lambda}$ three point functions are depicted in Figs(2). Using Feynman rules one could compute the divergent contributions from the graphs. As an example we calculate the one loop corrections to fig(2-a). It is given by:

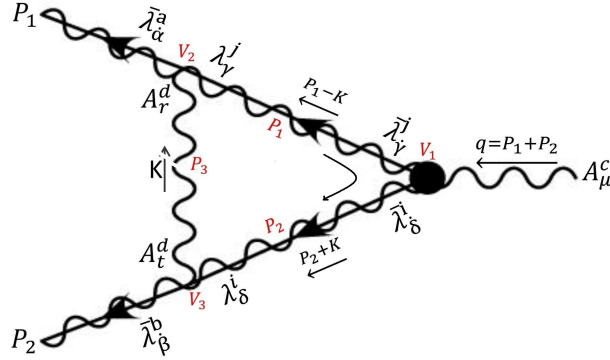


Figure 1: The diagram (2-a).

$$\begin{aligned} \Gamma_a^{\mu\dot{\alpha}\dot{\beta}} &= -g^2 \gamma^{AJI} d^{AJI} f^{BDJ} f^{CDI} C^{\mu\nu} \epsilon^{\dot{\gamma}\dot{\delta}} (p_1 + p_2)_\nu \bar{\sigma}^r{}^{\dot{\alpha}\gamma} \bar{\sigma}^t{}^{\dot{\beta}\delta} \sigma_{\gamma\dot{\gamma}}^m \sigma_{\delta\dot{\delta}}^n g_{rt} \\ &\times \int \frac{d^d k}{(2\pi)^d} \frac{(p_1 - k)_k (p_2 + k)_\lambda}{k^2 (p_1 - k)^2 (p_2 + k)^2} \end{aligned} \quad (31)$$

Using Feynman parameter in App. B:

$$\frac{1}{abc} = 2 \int_0^1 dx \int_0^1 dy \frac{x}{[axy + bx(1-y) + c(1-x)]^3}, \quad (32)$$

we can simplify denominator of Eq. (31)

$$\begin{aligned} \frac{1}{k^2(p_1 - k)^2(p_2 + k)^2} &= 2 \int_0^1 dx \int_0^1 dy \\ &\times \frac{x}{[k^2 + 2k \cdot [p_1 x(y-1) + p_2(1-x)] + p_1^2 x(1-y) + p_2^2(1-x)]^3}. \end{aligned} \quad (33)$$

By changing variables to

$$k' = k + p_1 x(y-1) + p_2(1-x),$$

the denominator of integral in Eq. (33) is given by:

$$\frac{1}{k^2(p_1 - k)^2(p_2 + k)^2} = 2 \int_0^1 dx \int_0^1 dy \frac{x}{[k^2 - \Delta]^3}$$

where

$$\Delta = [p_1 x(2y-1) + 2p_2(1-x)]^2$$

so, the integral of Eq. (32) is given by:

$$2 \int_0^1 dx \int_0^1 dy \int \frac{d^d k}{(2\pi)^d} \frac{x(p_1 - k)_k(p_2 + k)_\lambda}{[k^2 - \Delta]^3} \quad (34)$$

then we arrive:

$$2 \int_0^1 dx \int_0^1 dy \int \frac{d^d k}{(2\pi)^d} \frac{-k_k k_\lambda}{[k^2 - \Delta]^3} = \frac{-ig_{k\lambda}}{32\pi^2 \varepsilon} \quad (35)$$

we finally have for Eq. (31):

$$\Gamma_a^{\mu\dot{\alpha}\dot{\beta}} = 4iNL\gamma^{ABC}d^{ABC}d^A c^B c^C \epsilon^{\dot{\alpha}\dot{\beta}} C^{\mu\rho}(p_1 + p_2)_\rho \quad (36)$$

Moreover, as it be seen in Fig. (2-a), we have momentum - energy conserving in the loop:

$$q_\nu = (p_1 + p_2)_\nu$$

The divergent contributions up to one loop correction to diagrams in Fig. 2 are given by:

$$\begin{aligned}
\Gamma_{2-a}^{(1)\mu\dot{\alpha}\dot{\beta}} &= 4iNL\gamma^{ABC}d^{ABC}c^Ac^Bc^C\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\
\Gamma_{2-b}^{(1)\mu\dot{\alpha}\dot{\beta}} &= iNL\gamma^{ABC}d^{ABC}c^Ad^Bc^C\left[\frac{1}{2}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu + \frac{1}{3}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}(p_1 - p_2)_\nu\right] \\
&\quad + \text{one permutation} \\
\Gamma_{2-c}^{(1)\mu\dot{\alpha}\dot{\beta}} &= \frac{1}{4}iNL\gamma^{ABC}d^{ABC}c^Ad^Bc^C\left[\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}(4p_1 + 5p_2)_\nu - \frac{2}{3}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}(2p_1 + 7p_2)_\nu\right] \\
&\quad + \text{one permutation} \\
\Gamma_{2-d}^{(1)\mu\dot{\alpha}\dot{\beta}} &= -3iNL\gamma^{ABC}d^{ABC}c^A\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\
\Gamma_{2-e}^{(1)\mu\dot{\alpha}\dot{\beta}} &= -\frac{1}{2}iNL\gamma^{ABC}d^{ABC}c^Ad^Bc^C\left[\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} - 2(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}\right]p_{2\nu} \\
&\quad + \text{one permutation}
\end{aligned} \tag{37}$$

$Y^{\mu\nu}$ is been defined,

$$(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}} = \epsilon^{\dot{\alpha}\dot{\theta}}C^{\mu\rho}g_{\rho\lambda}(\bar{\sigma}^{\lambda\nu})_{\dot{\theta}}^{\dot{\beta}} \tag{38}$$

Where tensor $Y^{\mu\nu}$ is symmetric respect to both Lorentz and spinor indices and tensor $C^{\mu\nu}$ is anti-symmetric. In our computation permutations has taken into account by changing the position of C as well as symmetry factors. Adding the different divergent contributions from the diagrams of fig 2 corresponding to different $U(1)$ and $SU(N)$ parts, we have:

$$\begin{aligned}
\Sigma_{i=a}^e \Gamma_{2-i}^{(1)\mu\dot{\alpha}\dot{\beta}} &= \frac{15}{4}iNL\gamma^{abc}d^{abc}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu + 8iNL\gamma^{0bc}d^{0bc}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\
&\quad - \frac{1}{2}iNL\gamma^{a0c}d^{a0c}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}p_{2\nu} - \frac{1}{2}iNL\gamma^{ab0}d^{ab0}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}p_{1\nu} \\
&\quad + \frac{1}{2}iNL\gamma^{abc}d^{abc}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}(p_1 - p_2)_\nu \\
&\quad - iNL\gamma^{a0c}d^{a0c}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}p_{2\nu} + iNL\gamma^{ab0}d^{ab0}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}p_{1\nu}
\end{aligned} \tag{39}$$

Let us now continue with the relevant diagrams containing only C -deformed vertex and contributing to the four point functions (Fig. 3 and Fig. 4). Using the

Feynman rules, and considering all permutations between the same fields, the final result for 1PI graphs of Fig. 3 are given by:

$$\begin{aligned}
\Gamma_{3-a}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} &= NLg[\gamma^{EAB}d^{ABE}f^{CDE} - \gamma^{CDE}f^{ABE}d^{CDE}]d^A c^B c^C c^D \\
&\quad [\frac{1}{4}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} - \frac{1}{6}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}] + \text{Three permutations} \\
\Gamma_{3-b}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} &= NLg[\gamma^{EAC}d^{ACE}f^{BDE} + \gamma^{BDE}f^{ACE}d^{BDE}]d^A c^B c^C c^D \\
&\quad [\frac{1}{8}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} - \frac{1}{4}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}] + \text{One permutations} \\
\Gamma_{3-c}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} &= 2NLg\gamma^{EAB}d^{ABE}f^{CDE}c^A c^B \epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} \\
\Gamma_{3-d}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} &= NLg[\gamma^{EAD}d^{ADE}f^{BCE} + \gamma^{BCE}f^{ADE}d^{BCE}]d^A c^B c^C c^D \\
&\quad [\frac{1}{8}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} - \frac{5}{12}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}] + \text{One permutations} \\
\Gamma_{3-e}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} &= -NLg[\gamma^{EAB}d^{ABE}f^{CDE} - \gamma^{CDE}f^{ABE}d^{CDE}]d^A c^B c^C c^D \\
&\quad [\frac{1}{4}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} + \frac{7}{12}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}] + \text{Three permutations} \\
\Gamma_{3-f}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} &= NLg[\gamma^{EAB}d^{ABE}f^{CDE} + \gamma^{CDE}f^{ABE}d^{CDE}]d^A c^B c^C c^D \\
&\quad [\frac{3}{16}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} - \frac{3}{8}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}] + \text{One permutations} \\
\Gamma_{3-g}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} &= NLg[\frac{3}{8}\gamma^{EAB}d^{ABE}f^{CDE}d^A c^B \epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} + \frac{1}{4}\gamma^{CDE}f^{ABE}d^{CDE}c^C c^D(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}] \\
&\quad + \text{One permutations} \\
\Gamma_{3-h}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} &= -\frac{3}{2}NLg\gamma^{EAB}d^{ABE}f^{CDE}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} \\
\Gamma_{3-i}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} &= \frac{3}{4}NLg\gamma^{EAB}d^{ABE}f^{CDE}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}
\end{aligned} \tag{40}$$

Considering all diagrams the final result for Fig. 3 is given by:

$$\begin{aligned}
\Sigma_{i=a}^i \Gamma_{3-i}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} &= \frac{11}{4}NLg\gamma^{eab}d^{abe}f^{cde}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} + Ng\gamma^{ea0}d^{a0e}f^{cde}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} \\
&\quad + \frac{1}{2}NLg\gamma^{ecd}d^{cde}f^{abe}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}
\end{aligned} \tag{41}$$

Finally, The final divergence contributions(Fig. 4) which come from the last term containing $|C|^2 (\bar{\lambda}\bar{\lambda})^2$ are given by:

$$\Gamma_{4-a}^{(1)\dot{\alpha}\dot{\beta}\dot{\delta}\dot{\gamma}} = iL\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\delta}\dot{\gamma}}|C|^2 [N\gamma_0 d^{abe}d^{cde} + 2\gamma_0 d^{abcd} + 4\gamma_1]$$

$$\begin{aligned}
\Gamma_{4-b}^{(1)\dot{\alpha}\dot{\beta}\dot{\delta}\dot{\gamma}} &= iL\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\delta}\dot{\gamma}} |C|^2 \left[\frac{N}{2}\gamma_0 d^{abe} d^{cde} - 2\gamma_2 \right] \\
\Gamma_{4-c}^{(1)\dot{\alpha}\dot{\beta}\dot{\delta}\dot{\gamma}} &= iL\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\delta}\dot{\gamma}} |C|^2 \left[-\frac{N}{2}\gamma_0 d^{abe} d^{cde} + 2\gamma_2 \right] \\
\Gamma_{4-d}^{(1)\dot{\alpha}\dot{\beta}\dot{\delta}\dot{\gamma}} &= iL\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\delta}\dot{\gamma}} |C|^2 \left[-2\gamma_0 d^{abcd} + \frac{1}{3}(\tilde{d}^{abcd} - \tilde{d}^{acdb}) + 3\gamma_2 \right] \quad (42)
\end{aligned}$$

Note that we have considered all permutations between the same fields and changing the position of C and $|C|^2$, and adds all divergences come from Fig. 4. The final result for 1PI graphs of Fig. 4 is given by:

$$\begin{aligned}
\Sigma_{i=a}^d \Gamma_{4-i}^{(1)\dot{\alpha}\dot{\beta}\dot{\delta}\dot{\gamma}} &= \frac{5}{4}iNL\gamma_0\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\delta}\dot{\gamma}} d^{abe} d^{cde} |C|^2 + 4iL\gamma_1\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\delta}\dot{\gamma}} |C|^2 \\
&\quad + 3iL\gamma_2\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\delta}\dot{\gamma}} |C|^2 \quad (43)
\end{aligned}$$

In order to renormalise the theory and remove the divergences one should rescale and redefine coupling constants. This procedure is equivalent to introduce of some counter-terms to the lagrangian. Therefore, we introduce bare couplings according to:

$$C_B^{\mu\nu} = Z_C C^{\mu\nu}, \quad |C|_B^2 = Z_{|C|^2} |C|^2, \quad (44)$$

$$\gamma_B^{ABC} = Z_{\gamma_{ABC}} \gamma^{ABC}, \quad \gamma_B^{ABCDE} = Z_{\gamma_{ABCDE}} \gamma^{ABCDE}, \quad (45)$$

However in the language of counter-terms, a theory is renormalisable if the counter-terms are of the same form as those appearing in the original lagrangian(these counter terms are required to cancel the divergences). If we look at three and four point functions we see that the anomaly term

$$\gamma^{ABC} d^{ABC} (Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}, \quad g\gamma^{ECD} d^{CDE} f^{ABE} (Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}$$

called Y term is problematic because we can not add some kinds of counter-terms which cancel Y term, or one can say these terms spoil renormalisation. In the next section we add some extra lagrangian to the theory and prove that the NAC pure gauge theory is renormalisable.

4 Renormalization of $N = \frac{1}{2}$ deformed lagrangian

In this section we shall renormalise the theory and remove the divergences. In order to renormalise the theory we should add extra term (L_{Extra}) to the original lagrangian. The extra lagrangian is considered as follow:

$$\begin{aligned}
L_{Extra} = & +\frac{i}{16}d^{ABC}\kappa^{BAC}C^{\mu\nu}(\partial_\mu A_\nu^A - \partial_\nu A_\mu^A)\bar{\lambda}^B\bar{\lambda}^C \\
& -\frac{i}{4}g\kappa^{EDB}d^{BDE}f^{ACE}C^{\mu\nu}A_\mu^CA_\nu^D\bar{\lambda}^A\bar{\lambda}^B \\
& +\frac{i}{4}\kappa^{BAC}d^{ABC}A_\mu^A(\partial_\nu\bar{\lambda}^BY^{\mu\nu}\bar{\lambda}^C - \bar{\lambda}^BY^{\mu\nu}\partial_\nu\bar{\lambda}^C) \\
& +\frac{i}{2}g\kappa^{EDB}f^{ACE}d^{BDE}A_\mu^CA_\nu^D\bar{\lambda}^AY^{\mu\nu}\bar{\lambda}^B
\end{aligned} \tag{46}$$

The two last terms in Eq. (46) help us to make renormalisable NAC $U(N)$ gauge theory, the first two terms are needed because of gauge transformation rules. These terms are absent from the original lagrangian because of λ redefinition in Ref [1] as we shall explain in next part. Adding the above terms to original lagrangian, the total lagrangian is given by:

$$L_{total} = L_{original} + L_{Extra} \tag{47}$$

Since we add some terms to original lagrangian, we have to modify the gauge transformation and SUSY transformation. It is easy to show that L_{total} is preserved under following gauge transformation in $U(N)$ group:

$$\delta_\phi A_\mu^A = -2\partial_\mu\phi^A - f^{ABC}\phi^BA_\mu^C, \tag{48}$$

$$\delta_\phi\bar{\lambda}_\alpha^A = -f^{ABC}\phi^B\bar{\lambda}_\alpha^C \tag{49}$$

$$\delta_\phi\lambda_\alpha^A = -f^{ABC}\phi^B - \frac{1}{2}\kappa^{ABC}d^{ABC}C^{\mu\nu}\sigma_{\nu\alpha\dot{\alpha}}\partial_\mu\phi^B\bar{\lambda}^{\dot{\alpha}C} \tag{50}$$

$$\delta_\phi D^A = -f^{ABC}\phi^BD^C \tag{51}$$

Where κ^{ABC} is considered as arbitrary coupling which depends on A, B, C values. The gauge transformation is not canonical because the transformation of λ depends

on the non- anti- commutative parameter C . However; the SUSY transformation does not change except

$$\delta\lambda_\alpha^A = i\epsilon_\alpha D^A + (\sigma^{\mu\nu}\epsilon)_\alpha [F_{\mu\nu}^A + \frac{1}{2}iC_{\mu\nu}(\gamma^{ABC} + \frac{1}{2}\kappa^{ABC})d^{ABC}\bar{\lambda}^B\bar{\lambda}^C]$$

So yet we can refer to eqs. (21-23). In our work in order to renormalise the NAC $SU(N) \times U(1)$ gauge theory, we choose

$$\kappa^{ABC} = \xi\gamma^{BAC}c^Ac^Bd^C \quad (52)$$

here ξ is considered as a coefficient. Then the extra lagrangian is given by:

$$\begin{aligned} L_{extra} = & \frac{i}{16}\kappa_1 d^{abc}C^{\mu\nu}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)\bar{\lambda}^b\bar{\lambda}^c - \frac{i}{8}\kappa_1 g f^{cde}d^{abe}C^{\mu\nu}A_\mu^c A_\nu^d \bar{\lambda}^a\bar{\lambda}^b \\ & + \frac{i}{4}\kappa_1 d^{abc}(\partial_\mu \bar{\lambda}^b Y^{\mu\nu}\bar{\lambda}^c - \bar{\lambda}^b Y^{\mu\nu}\partial_\mu \bar{\lambda}^c)A_\nu^a - \frac{i}{4}\kappa_1 g f^{abe}d^{cde}A_\mu^c A_\nu^d \bar{\lambda}^a Y^{\mu\nu}\bar{\lambda}^b \\ & + \frac{i}{16}\kappa_3 d^{ab0}C^{\mu\nu}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)\bar{\lambda}^b\bar{\lambda}^0 + \frac{i}{4}\kappa_3 d^{ab0}(\partial_\mu \bar{\lambda}^b Y^{\mu\nu}\bar{\lambda}^0 - \bar{\lambda}^b Y^{\mu\nu}\partial_\mu \bar{\lambda}^0)A_\nu^a \\ & - \frac{i}{4}\kappa_3 g f^{cde}d^{0be}C^{\mu\nu}A_\mu^c A_\nu^d \bar{\lambda}^0\bar{\lambda}^b, \end{aligned} \quad (53)$$

where

$$\kappa_1 = \kappa^{abc} = \xi\gamma^{abc}, \quad \kappa_3 = \kappa^{ab0} = 2\xi\gamma^{ab0}, \quad \kappa^{0ab} = \kappa^{a0b} = 0. \quad (54)$$

In according to Eq.(46), the Y terms leads to new interactions hence, we have to consider new vertices in 1PI graphs. It means we display these interactions that have been hidden. So, we should calculate 1PI diagrams considering new vertices or we should modify vertices. Finally, we find that theory is renormalisable.

In this case the new action is given by:

$$\begin{aligned} S_{total} = & \int d^4x \left[-\frac{1}{4}F^{\mu\nu A}F_{\mu\nu}^A - i\bar{\lambda}^A\bar{\sigma}^\mu(D_\mu\lambda)^A + \frac{1}{2}D^A D^A - \frac{1}{2}id^{ABC}\gamma^{ABC}C^{\mu\nu}F_{\mu\nu}^A\bar{\lambda}^B\bar{\lambda}^C \right. \\ & \left. + \frac{1}{8}|C|^2 d^{ABE}d^{CDE}\gamma^{ABCDE}(\bar{\lambda}^A\bar{\lambda}^B)(\bar{\lambda}^C\bar{\lambda}^D) + L_{Extra} \right]. \end{aligned} \quad (55)$$

We have to calculate new (1PI) diagrams contributing to the new terms(those containing both parameter C and Y) which are depicted in Figs. 5 and 6. The

results for the new graphs contributing to the new interaction terms in Eq. (53) are the same as the C terms so we shall not give detailed results. For example in order to calculate Fig. 5-a, we should only change the NAC vertex $i\gamma^{AJI}d^{AJI}C^{\mu\nu}\epsilon^{\dot{\gamma}\dot{\delta}}(p_1+p_2)_\nu$ in eq. (31) to $-\frac{i}{8}\kappa^{JAI}d^{AJI}C^{\mu\nu}\epsilon^{\dot{\gamma}\dot{\delta}}q_\nu + \frac{i}{4}\kappa^{JAI}d^{AJI}(Y^{\mu\nu})^{\dot{\gamma}\dot{\delta}}(p_1-p_2-2k)_\nu$.

The result for the graphs in Fig. 5-a is given by:

$$\Gamma_{5-a-Extra}^{(1)\mu\dot{\alpha}\dot{\beta}} = \frac{i}{4}NL\kappa^{BAC}d^{ABC}d^Ac^Bc^C\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \quad (56)$$

Beside, the total divergent contribution for new graphs in Fig. 5 is given by:

$$\begin{aligned} \Gamma_{1PI-Extra\ graph}^{(1)\mu\dot{\alpha}\dot{\beta}} &= -\frac{i}{2}NL\kappa^{abc}d^{abc}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\ &\quad + iNL\kappa^{abc}Ld^{abc}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}(p_1-p_2)_\nu \end{aligned} \quad (57)$$

In order to obtain divergent contribution for 1PI graphs with both C and Y parameters, one should add results of Eq. (39) and Eq. (56)

$$\begin{aligned} \Gamma_{1PI-total}^{(1)\mu\dot{\alpha}\dot{\beta}} &= (-\frac{1}{2}\kappa^{abc} + \frac{15}{4}\gamma^{abc})iNLd^{abc}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu + 8\gamma^{0bc}iNLd^{0bc}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\ &\quad - \frac{1}{2}\gamma^{a0c}NLd^{a0c}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu - \frac{1}{2}\gamma^{ab0}NLd^{ab0}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\ &\quad + (\kappa^{abc} + \frac{1}{2}\gamma^{abc})iNLd^{abc}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}(p_1-p_2)_\nu \\ &\quad - \gamma^{a0c}iNLd^{a0c}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}p_{2\nu} + \gamma^{ab0}iNLd^{ab0}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}p_{1\nu} \end{aligned} \quad (58)$$

There are new graphs(Fig. 6) for one loop corrections of the four point function. The total contributions as different $SU(N) \times U(1)$ parts corresponds to Fig. 5 is given by:

$$\begin{aligned} \Gamma_{1PI-Extra\ graphs}^{\mu\nu\dot{\alpha}\dot{\beta}} &= -\frac{3}{4}NL\kappa^{edb}gd^{abe}f^{cde}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} - \frac{1}{2}NL\kappa^{ed0}gd^{0de}f^{ace}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} \\ &\quad + \frac{3}{2}NL\kappa^{ecd}gd^{cde}f^{abe}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}} \end{aligned} \quad (59)$$

Then, the total four point 1PI divergent contribution is given by:

$$\Gamma_{1PI-total}^{\mu\nu\dot{\alpha}\dot{\beta}} = (3\kappa_1 + \frac{11}{4}\gamma^{eab})NLgd^{abe}f^{cde}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}$$

$$\begin{aligned}
& +\gamma^{ea0}NLgd^{a0e}f^{cde}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} \\
& -\frac{1}{2}\kappa_3NLgd^{0be}f^{cde}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} \\
& +(\frac{3}{2}\kappa_1+\frac{1}{2}\gamma^{ecd})NLgd^{cde}f^{abe}(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}
\end{aligned} \tag{60}$$

Fortunately, new terms does not effect on four point function is containing $|C|^2$ $(\bar{\lambda}\bar{\lambda})^2$. In order to compute counter terms we should decompose the lagrangian to the $SU(N) \times U(1)$ parts because some interaction terms such as term which correspond to $(U(1))^3$ receive no quantum corrections. It is given by:

$$\begin{aligned}
L_{total} = & -\frac{1}{2}id^{abc}(\gamma^{abc}-\frac{1}{8}\kappa_1)C^{\mu\nu}(\partial_\mu A_\nu^a-\partial_\nu A_\mu^a)\bar{\lambda}^b\bar{\lambda}^c \\
& -\frac{1}{2}id^{000}(\gamma^{000}-0)C^{\mu\nu}(\partial_\mu A_\nu^0-\partial_\nu A_\mu^0)\bar{\lambda}^0\bar{\lambda}^0 \\
& -\frac{1}{2}id^{a0c}(\gamma^{a0c}-0)C^{\mu\nu}(\partial_\mu A_\nu^a-\partial_\nu A_\mu^a)\bar{\lambda}^0\bar{\lambda}^c \\
& -\frac{1}{2}id^{ab0}(\gamma^{ab0}-\frac{1}{8}\kappa_3)C^{\mu\nu}(\partial_\mu A_\nu^a-\partial_\nu A_\mu^a)\bar{\lambda}^b\bar{\lambda}^0 \\
& -\frac{1}{2}id^{0bc}(\gamma^{0bc}-0)C^{\mu\nu}(\partial_\mu A_\nu^0-\partial_\nu A_\mu^0)\bar{\lambda}^b\bar{\lambda}^c \\
& +\frac{1}{2}ig(\gamma^{eab}-\frac{1}{4}\kappa_1)d^{abe}f^{cde}C^{\mu\nu}A_\mu^cA_\nu^d\bar{\lambda}^a\bar{\lambda}^b \\
& +\frac{1}{2}ig(\gamma^{e0b}-\frac{1}{2}\kappa_3)d^{0be}f^{cde}C^{\mu\nu}A_\mu^cA_\nu^d\bar{\lambda}^0\bar{\lambda}^b \\
& +\frac{1}{2}ig(\gamma^{ea0}-0)d^{a0e}f^{cde}C^{\mu\nu}A_\mu^cA_\nu^d\bar{\lambda}^a\bar{\lambda}^0 \\
& +\frac{1}{8}\gamma_0|C|^2d^{abe}d^{cde}(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d) \\
& +\frac{1}{4N}\gamma_1|C|^2(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^b\bar{\lambda}^b)+\frac{1}{N}\gamma_2|C|^2(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^0\bar{\lambda}^0) \\
& +\frac{i}{4}\kappa_1d^{abc}A_\mu^a(\partial_\nu\bar{\lambda}^bY^{\mu\nu}\bar{\lambda}^c-\bar{\lambda}^bY^{\mu\nu}\partial_\nu\bar{\lambda}^c) \\
& +\frac{i}{4}\kappa_3d^{ab0}A_\mu^a(\partial_\nu\bar{\lambda}^bY^{\mu\nu}\bar{\lambda}^0-\bar{\lambda}^bY^{\mu\nu}\partial_\nu\bar{\lambda}^0) \\
& -\frac{i}{4}\kappa_1gf^{abe}d^{cde}A_\mu^cA_\nu^d\bar{\lambda}^aY^{\mu\nu}\bar{\lambda}^b
\end{aligned} \tag{61}$$

The C and Y -dependent part of the action are renormalisable if one introduces bare fields and couplings according to Eqs. (26) and Eqs. (44, 45) as well as:

$$Y_B^{\mu\nu} = Z_Y Y^{\mu\nu}, \kappa_B^{ABC} = Z_\xi Z_{\gamma BAC} \kappa^{ABC} \tag{62}$$

Then in order to find some counter terms we have to introduce the following identities:

$$\begin{aligned}
Z_{\gamma abc} Z_C Z_A^{1/2} Z_\lambda &= \delta_{01} + 1, \quad Z_{\kappa_1} Z_C Z_A^{1/2} Z_\lambda = \delta_1 + 1, \\
Z_{\gamma^{000}} Z_C Z_{A^0}^{1/2} Z_{\lambda^0} &= \delta_{02} + 1, \\
Z_{\gamma^{a0c}} Z_C Z_A^{1/2} Z_\lambda^{1/2} Z_{\lambda^0}^{1/2} &= \delta_{03} + 1, \\
Z_{\gamma^{ab0}} Z_C Z_A^{1/2} Z_\lambda^{1/2} Z_{\lambda^0}^{1/2} &= \delta_{04} + 1, \quad Z_{\kappa_3} Z_C Z_A^{1/2} Z_\lambda^{1/2} Z_{\lambda^0}^{1/2} = \delta_4 + 1, \\
Z_{\gamma^{0bc}} Z_C Z_{A^0}^{1/2} Z_\lambda &= \delta_{05} + 1, \\
Z_g Z_{\gamma abc} Z_C Z_A Z_\lambda &= \delta_{06} + 1, \quad Z_g Z_{\kappa_1} Z_C Z_A Z_\lambda = \delta_6 + 1, \\
Z_g Z_{\gamma^{e0b}} Z_C Z_A Z_{\lambda^0}^{1/2} Z_\lambda^{1/2} &= \delta_{07} + 1, \quad Z_g Z_{\kappa_3} Z_C Z_A Z_{\lambda^0}^{1/2} Z_\lambda^{1/2} = \delta_7 + 1, \\
Z_g Z_{\gamma^{ea0}} Z_C Z_A Z_\lambda^{1/2} Z_{\lambda^0}^{1/2} &= \delta_{08} + 1, \\
Z_{\gamma^0} Z_{|C|^2} Z_\lambda^2 &= \delta_{09} + 1, \quad Z_{\gamma^1} Z_{|C|^2} Z_\lambda^2 = \delta_{010} + 1, \\
Z_{\gamma^2} Z_{|C|^2} Z_\lambda Z_{\lambda^0} &= \delta_{011} + 1, \\
Z_{\kappa_1} Z_Y Z_A^{1/2} Z_\lambda &= \delta_{12} + 1, \quad Z_{\kappa_3} Z_Y Z_A^{1/2} Z_\lambda^{1/2} Z_{\lambda^0}^{1/2} = \delta_{13} + 1, \\
Z_g Z_{\kappa_1} Z_Y Z_A Z_\lambda &= \delta_{14} + 1,
\end{aligned} \tag{63}$$

Adding the following counter-term terms to the part of the total action and comparing the expression with the bare action, the theory should be renormalisable.

The full C dependent part of the action can be written as:

$$\begin{aligned}
S = \int d^4x & \left[-\frac{1}{2} i d^{abc} (\gamma^{abc} (1 + \delta_{01}) - \frac{1}{8} \kappa_1 (1 + \delta_1)) C^{\mu\nu} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) \bar{\lambda}^b \bar{\lambda}^c \right. \\
& - \frac{1}{2} i d^{000} (\gamma^{000} (1 + \delta_{02}) - 0) C^{\mu\nu} (\partial_\mu A_\nu^0 - \partial_\nu A_\mu^0) \bar{\lambda}^0 \bar{\lambda}^0 \\
& - \frac{1}{2} i d^{a0c} (\gamma^{a0c} (1 + \delta_{03}) - 0) C^{\mu\nu} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) \bar{\lambda}^0 \bar{\lambda}^c \\
& - \frac{1}{2} i d^{ab0} (\gamma^{ab0} (1 + \delta_{04}) - \frac{1}{8} \kappa_3 (1 + \delta_4)) C^{\mu\nu} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) \bar{\lambda}^b \bar{\lambda}^0 \\
& - \frac{1}{2} i d^{0bc} (\gamma^{0bc} (1 + \delta_{05}) - 0) C^{\mu\nu} (\partial_\mu A_\nu^0 - \partial_\nu A_\mu^0) \bar{\lambda}^b \bar{\lambda}^c \\
& + \frac{1}{2} i g (\gamma^{eab} (1 + \delta_{06}) - \frac{1}{4} \kappa_1 (1 + \delta_6)) d^{abe} f^{cde} C^{\mu\nu} A_\mu^c A_\nu^d \bar{\lambda}^a \bar{\lambda}^b \\
& \left. + \frac{1}{2} i g (\gamma^{e0b} (1 + \delta_{07}) - \frac{1}{2} \kappa_3 (1 + \delta_7)) d^{0be} f^{cde} C^{\mu\nu} A_\mu^c A_\nu^d \bar{\lambda}^0 \bar{\lambda}^b \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} i g (\gamma^{ea0} (1 + \delta_{08}) - 0) d^{a0e} f^{cde} C^{\mu\nu} A_\mu^c A_\nu^d \bar{\lambda}^a \bar{\lambda}^0 \\
& + \frac{1}{8} (1 + \delta_{09}) \gamma_0 |C|^2 d^{abe} d^{cde} (\bar{\lambda}^a \bar{\lambda}^b) (\bar{\lambda}^c \bar{\lambda}^d) \\
& + \frac{1}{4N} (1 + \delta_{010}) \gamma_1 |C|^2 (\bar{\lambda}^a \bar{\lambda}^a) (\bar{\lambda}^b \bar{\lambda}^b) + \frac{1}{N} (1 + \delta_{011}) \gamma_2 |C|^2 (\bar{\lambda}^a \bar{\lambda}^a) (\bar{\lambda}^0 \bar{\lambda}^0) \\
& + \frac{i}{4} (1 + \delta_{12}) \kappa_1 d^{abc} A_\mu^a (\partial_\nu \bar{\lambda}^b Y^{\mu\nu} \bar{\lambda}^c - \bar{\lambda}^b Y^{\mu\nu} \partial_\nu \bar{\lambda}^c) \\
& + \frac{i}{4} (1 + \delta_{13}) \kappa_3 d^{ab0} A_\mu^a (\partial_\nu \bar{\lambda}^b Y^{\mu\nu} \bar{\lambda}^0 - \bar{\lambda}^b Y^{\mu\nu} \partial_\nu \bar{\lambda}^0) \\
& - \frac{i}{4} (1 + \delta_{14}) \kappa_1 g f^{abe} d^{cde} A_\mu^c A_\nu^d \bar{\lambda}^a Y^{\mu\nu} \bar{\lambda}^b \Big].
\end{aligned} \tag{64}$$

where in order to renormalise the C -dependent part of the action we have to obtain the values of δ_i by solving the following equations:

$$\Gamma_{1PI-total}^{(1)\mu\dot{\alpha}\dot{\beta}} + \Gamma_{C.T}^{\mu\dot{\alpha}\dot{\beta}} = 0, \tag{65}$$

$$\Gamma_{1PI-total}^{(1)\mu\nu\dot{\alpha}\dot{\beta}} + \Gamma_{C.T}^{\mu\nu\dot{\alpha}\dot{\beta}} = 0, \tag{66}$$

$$\Gamma_{1PI-total}^{(1)\dot{\alpha}\dot{\beta}\dot{\delta}\dot{\gamma}} + \Gamma_{C.T}^{\dot{\alpha}\dot{\beta}\dot{\delta}\dot{\gamma}} = 0, \tag{67}$$

Where $\Gamma_{C.T}$ s come from counter terms (Appendix E). Then, using Z_g , Z_A , Z_λ for $SU(N)$ sector, and Z_{g_0} , Z_{A^0} , Z_{λ^0} for $U(1)$ sector, we obtain the renormalisation constants as :

$$Z_\xi^{(1)} = -\frac{2}{\xi} NL, \quad Z_h^{(1)} = -NL \tag{68}$$

As it is understood because ξ is renormalised, so κ^{ABC} should be renormalised.

Moreover we obtain

$$Z_C = Z_Y = Z_{|C|^2} = 1 \tag{69}$$

$$(Z_C)^2 = Z_{|C|^2} = 1 \tag{70}$$

It is found that our result is compatible with Ref [11, 13]. Of course a natural expectation would be that $Z_C = Z_Y$, because of Eq. (36) we know that $Y \propto C$.

We demonstrate that the theory is renormalisable and the $N = 1/2$ supersymmetric as well as NAC $SU(N) \times U(1)$ pure gauge theory is preserved. This point also

has been concluded in Ref [11] and suggested in Ref [6], where was supposed to be correct to all orders. We also arrive at the conclusion that it is not needed to renormalise the non-anticommutativity parameter C . Beside our full lagrangian is the same form as derived from non-anticommutative superspace, however $Z_{\kappa ABC}$ and $Z_{\gamma ABC}$ depends on whether A, B, C are $SU(N)$ indices or $U(1)$ indices. It seems to imply that the renormalised theory is not $U(N)$ non-anticommutative theory any more. Because the $U(N)$ structure is broken by renormalisation.

In order to clarify L_{Extra} we redefine the component λ as

$$\lambda^A \longrightarrow \lambda^A - \frac{1}{4} \kappa^{ABC} d^{ABC} C^{\mu\nu} A_\mu^B \sigma_\nu \bar{\lambda}^C \quad (71)$$

Then, put it in Eq. (18), and obtain Eq. (55) (in other words L_{total} is result of λ redefinition in $L_{original}$). The λ redefinition just affects the gaugino kinetic term. Our redefinition is opposite to Refs [1, 20]. They have redefined λ in order to make gauge transformations be canonical; however it causes theory unrenormalisable because in that case some terms are been hidden in the lagrangian. In order to reverse process we should add hidden terms by hand or come back to original definition of λ Eq. (71); however we lose gauge canonical transformation. Beside, because κ^{ABC} is obtained renormalised, the redefinition of λ^A is called $\hat{\lambda}^A$ should be renormalised. Finally divergent field redefinition in Ref [11] is reinterpreted as renormalised $\hat{\lambda}^A$. Our results show it is not needed to deform the classical action if one do not use the field redefinition of Ref [1].

4.1 discussion on $\xi \longrightarrow 0$

It is worthwhile to investigate our results in the case of limit $\xi \longrightarrow 0$. Indeed in this case, 1PI graphs from new terms are vanished. We would like to present this claim as follow. Instance we know that $\Gamma_{1-Extra}^{(1)\mu\dot{\alpha}\dot{\beta}}(\xi)$ comes from extra lagrangian.

Then total divergent contribution for three point function would be finite if we add some counter terms as follow:

$$\Gamma_{1PI-total}^{(1)\mu\dot{\alpha}\dot{\beta}} = \Gamma_{1-original}^{(1)\mu\dot{\alpha}\dot{\beta}} + \Gamma_{1-Extra}^{(1)\mu\dot{\alpha}\dot{\beta}}(\xi) + \Gamma_{C.T}^{\mu\dot{\alpha}\dot{\beta}} = 0 \quad (72)$$

we expect to obtain $Z_C = Z_Y = Z_{|C|^2} = 1$ (it means the NAC structure would be preserved under the procedure of renormalisation). Then, the above equation leads to:

$$\delta_1 = -\left(\frac{2}{\xi} + 4\right)NL, \quad (73)$$

$$\delta_{02} = 0, \quad (74)$$

$$\delta_{03} = \delta_{04} = 0, \quad (75)$$

$$\delta_4 = -\frac{2}{\xi}NL, \quad (76)$$

$$\delta_{05} = -8NL \quad (77)$$

$$\delta_9 = -\left(\frac{2}{\xi} + 4\right)NL, \quad (78)$$

$$\delta_{11} = -\frac{2}{\xi}NL \quad (79)$$

Note that second terms in eqs. (73) and (78) are related to divergent contribution from new 1PI graphs. In order to find Z_ξ we have to benefit from eqs. (63). Hence Z_ξ up to one order is given by:

$$Z_\xi = \left(1 - \frac{2}{\xi}NL - 4NL\right)(1 + 4NL) = 1 - \frac{2}{\xi}NL, \quad (80)$$

Now, in the case of $\xi \rightarrow 0$ eqs. (73-79) result to

$$\delta_1 = -\frac{2}{\xi}NL, \quad \delta_{02} = 0, \quad \delta_{03} = \delta_{04} = 0, \quad (81)$$

$$\delta_4 = -\frac{2}{\xi}NL, \quad \delta_{05} = -8NL, \quad \delta_9 = -\frac{2}{\xi}NL, \quad \delta_{11} = -\frac{2}{\xi}NL. \quad (82)$$

In fact we have neglected the divergent contributions from new graphs, however; Z_ξ has not been changed. This event could be checked for four point function as

well. In this case we obtain the results of Ref [11, 10], but because $Z_\xi \neq 0$ it is easy to show it is not necessary to define a nonlinear divergence field redefinition. If we consider

$$\hat{\lambda}^A = \lambda^A - \frac{1}{4}\kappa^{ABC}d^{ABC}C^{\mu\nu}A_\mu^B\sigma_\nu\bar{\lambda}^C \quad (83)$$

Then the variation of λ^A is written by

$$\delta\lambda^A = -\frac{1}{4}\kappa^{ABC}d^{ABC}C^{\mu\nu}A_\mu^B\sigma_\nu\bar{\lambda}^C \quad (84)$$

Consequently, after the procedure of renormalization, the nonlinear divergent field redefinition in Ref [11] automatically is generated:

$$\delta\lambda^A = \frac{1}{2}NL\gamma^{BAC}c^Ac^Bd^Cd^{ABC}C^{\mu\nu}A_\mu^B\sigma_\nu\bar{\lambda}^C \quad (85)$$

5 Conclusion

We have computed 1pI corrections for the pure $N = \frac{1}{2}$ supersymmetric $SU(N) \times U(1)$ gauge theory at one loop order. We have proved the theory is renormalisable up to one loop order using a standard way of renormalisation if one adds some new terms to the original lagrangian. We have shown it is possible to interrupt these are hidden terms because of λ redefinition. It is worth to investigate if it is possible to show that the problems which arise in renormalisation of $N = \frac{1}{2}$ supersymmetric theories come from the redefined vector superfield.

We have shown there is no need to define divergent redefinition of λ . Moreover we suggest all works which have been done based on divergent field redefinition should be reviewed. We have used the $N = \frac{1}{2}$ $U(N)$ gauge group action because as discussed in [21], just non-anticommutative theory with $U(N)$ gauge group is well-defined. Moreover it is worth to investigate the renormalization of theory at higher loops or including chiral matter in the standard form of renormalisation

method. We guess the problem of renormalisation of non-anticommutative theory at component formalism is because of λ redefinition in [1].

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A : New Algebra For Non Commutative Parameters C And Y

We have found the new properties for C and Y parameters that we have made frequent use in our calculations. $C^{\mu\nu}$ is related to the non-anti-commutativity parameter $C^{\alpha\beta}$ by:

$$C^{\mu\nu} = C^{\alpha\beta} \epsilon_{\beta\gamma} \sigma_{\alpha}^{\mu\nu \gamma} \quad (86)$$

also, we have:

$$C^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\gamma} \sigma_{\gamma}^{\mu\nu \beta} C_{\mu\nu}, \quad (87)$$

where

$$C^{\mu\nu} \sigma_{\nu\alpha\dot{\beta}} = \epsilon_{\alpha\beta} C^{\beta\gamma} \sigma_{\gamma\dot{\beta}}^{\mu}, \quad (88)$$

$$C^{\mu\nu} \bar{\sigma}_{\nu}^{\dot{\alpha}\beta} = C^{\beta\alpha} \epsilon_{\alpha\gamma} \bar{\sigma}^{\mu\dot{\alpha}\gamma} \quad (89)$$

we have used the following notations:

$$C^{\alpha\gamma} \epsilon_{\gamma\beta} = C^{\alpha}{}_{\beta}, \quad (90)$$

$$\epsilon_{\beta\gamma} C^{\gamma\alpha} = C_{\beta}{}^{\alpha}, \quad (91)$$

$$C^{\alpha}{}_{\beta} = -(C_{\beta}{}^{\alpha})^T \quad (92)$$

where in last equation the symbol of T is used for transposed. Also for Y parameter we have:

$$(Y^{\mu\nu})_{\dot{\theta}}^{\dot{\alpha}} = C^{\mu\rho} g_{\rho\lambda} (\bar{\sigma}^{\lambda\nu})_{\dot{\theta}}^{\dot{\alpha}} , \quad (93)$$

$$(Y^{\mu\nu})_{\dot{\theta}}^{\dot{\alpha}} (Y_{\mu\nu})_{\dot{\gamma}}^{\dot{\beta}} = \frac{1}{4} [\delta_{\dot{\theta}}^{\dot{\alpha}} \delta_{\dot{\gamma}}^{\dot{\beta}} - 2\delta_{\dot{\gamma}}^{\dot{\alpha}} \delta_{\dot{\theta}}^{\dot{\beta}}] |C|^2 , \quad (94)$$

$$Tr[|Y|^2] = -\frac{3}{2} |C|^2 , \quad (95)$$

$$(Y^{\mu\nu})_{\dot{\theta}}^{\dot{\alpha}} \sigma_{\nu\gamma\dot{\gamma}} = \frac{1}{2} [-2C^{\alpha\theta} \bar{\sigma}^{\mu\dot{\alpha}\theta} \epsilon_{\alpha\gamma} \epsilon_{\dot{\theta}\dot{\gamma}} + C_{\gamma}^{\theta} \sigma^r_{\theta\dot{\gamma}} \delta_{\dot{\theta}}^{\dot{\alpha}}] , \quad (96)$$

$$(Y^{\mu\nu})_{\dot{\theta}}^{\dot{\alpha}} \bar{\sigma}_{\nu}^{\dot{\gamma}\gamma} = \frac{1}{2} C^{\gamma\theta} [-2\bar{\sigma}^{\mu\dot{\alpha}\theta} \delta_{\dot{\theta}}^{\dot{\gamma}} + \bar{\sigma}^{\mu\dot{\gamma}\theta} \delta_{\dot{\theta}}^{\dot{\alpha}}] , \quad (97)$$

$$(Y^{\mu\nu})_{\dot{\theta}}^{\dot{\alpha}} \bar{\sigma}_{\mu}^{\dot{\gamma}\gamma} \sigma_{\nu\delta\dot{\delta}} = C^{\gamma\delta} [2\epsilon^{\dot{\alpha}\dot{\gamma}} \epsilon_{\dot{\theta}\dot{\delta}} + \delta_{\dot{\delta}}^{\dot{\gamma}} \delta_{\dot{\theta}}^{\dot{\alpha}}] , \quad (98)$$

$$(Y^{\mu\nu})_{\dot{\theta}}^{\dot{\alpha}} (\bar{\sigma}_{\nu\mu})_{\dot{\delta}}^{\dot{\beta}} = 0 , \quad (99)$$

$$(Y^{\mu\nu})_{\dot{\theta}}^{\dot{\alpha}} g_{\mu\nu} = 0 , \quad (100)$$

$$(Y^{\mu\nu})_{\dot{\alpha}}^{\dot{\alpha}} = 0, \text{ (Y is traceless)}. \quad (101)$$

B : Feynman Parameters

$$\frac{1}{ab} = \int_0^1 dx \frac{1}{[(1-x)b + xa]^2} \quad (102)$$

$$\frac{1}{a^n b} = n \int_0^1 dx \frac{x^{n-1}}{[(1-x)b + xa]^{n+1}} \quad (103)$$

$$\begin{aligned} \frac{1}{abc} &= 2 \int_0^1 dx \int_0^1 dy \frac{x}{[axy + bx(1-y) + c(1-x)]^3} \\ &= 2 \int_0^1 du \int_0^{1-u} dw \frac{1}{[aw + b(1-u-w) + cu]^3} \end{aligned} \quad (104)$$

$$\begin{aligned} \frac{1}{a_1 a_2 \dots a_n} &= (n-1)! \int_0^1 dx_n dx_{n-1} \dots dx_2 \\ &\quad \times \frac{x_n^{n-2} x_{n-1}^{n-3} \dots x_3^1 x_2^0}{[(1-x_n)a_n + x_n[(1-x_{n-1})a_{n-1} + x_{n-1}[\dots + x_3[(1-x_2)a_2 + x_2 a_1]] \dots]^n} \end{aligned} \quad (105)$$

C : The d Dimensional Integrals In Minkowski Space

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - \Delta]^n} = \frac{(-1)^n i \Gamma(n - \frac{d}{2})}{(4\pi)^{\frac{d}{2}} \Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2}} \quad (106)$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^2}{[l^2 - \Delta]^n} = \frac{(-1)^{n-1} i}{(4\pi)^{\frac{d}{2}}} \frac{d}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 1} \quad (107)$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^\mu l^\nu}{[l^2 - \Delta]^n} = \frac{(-1)^{n-1} i}{(4\pi)^{\frac{d}{2}}} \frac{g^{\mu\nu}}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 1} \quad (108)$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{(l^2)^2}{[l^2 - \Delta]^n} = \frac{(-1)^n i}{(4\pi)^{\frac{d}{2}}} \frac{d(d+2)}{4} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 2} \quad (109)$$

$$\begin{aligned} \int \frac{d^d l}{(2\pi)^d} \frac{l^\mu l^\nu l^\rho l^\sigma}{[l^2 - \Delta]^n} &= \frac{(-1)^n i}{(4\pi)^{\frac{d}{2}}} \frac{\Gamma(n - \frac{d}{2} - 2)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n - \frac{d}{2} - 2} \\ &\times \frac{1}{4} (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}). \end{aligned} \quad (110)$$

D : Feynman Rules

Here, we collect Feynman rules would be used in order to calculate 1PI digrams for current theory. propagators for each field could be as follow:

Gauge field A_μ^A :

$$-\frac{i g_{\mu\nu}}{p^2},$$

Gaugino field λ_α^A :

$$\frac{i p_\mu \sigma_{\alpha\dot{\alpha}}^\mu}{p^2},$$

Auxiliary Boson field D^A : this field could not be propagated.

Scalar field F^A : this field could not be propagated.

Spinor field ψ_α^A :

$$\frac{i p_\mu \sigma_{\alpha\dot{\alpha}}^\mu}{p^2},$$

Scalar field ϕ^A :

$$\frac{i}{p^2}$$

Vertices comes from each interaction in the theory so, we have some vertices as follow:

three- Gauge coupling $A_\mu^a, A_\nu^b, A_\rho^c$ with momentum k, p, q respectively:

$$gf^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu],$$

four- Gauge coupling $A_\mu^a, A_\nu^b, A_\rho^c, A_\sigma^d$:

$$\begin{aligned} & -ig^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ & + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ & + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})] \end{aligned}$$

Gaugion $\bar{\lambda}_\alpha^A$ -Gaugino λ_α^C -Gauge A_μ^B vertex:

$$-gf^{ABC}\bar{\sigma}^{\mu\dot{\alpha}\alpha}$$

NAC in Gauge A_μ^A with momentum k_ν -Gaugino $\bar{\lambda}_\alpha^B$ -Gaugino $\bar{\lambda}_\beta^C$ vertex:

$$id^{ABC}\gamma^{ABC}C^{\mu\nu}\epsilon^{\dot{\alpha}\dot{\beta}}k_\nu$$

NAC in Gauge A_μ^C -Gauge A_ν^D -Gaugino $\bar{\lambda}_\alpha^A$ -Gaugino $\bar{\lambda}_\beta^B$ vertex:

$$-\frac{1}{2}gd^{ABE}\gamma^{EAB}f^{CDE}C^{\mu\nu}\epsilon^{\dot{\alpha}\dot{\beta}}$$

NAC in four Gauginos $(\bar{\lambda}_\alpha^A\bar{\lambda}_{\dot{\beta}}^B)(\bar{\lambda}_\gamma^C\bar{\lambda}_{\dot{\delta}}^D)$ vertex:

$$\frac{i}{8} |C|^2 d^{ABE}d^{CDE}\gamma^{ABCDE}\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}$$

For total lagrangian we can write

$$L_{total} = L_{C=0} + L_C + L_{Extra} \quad (111)$$

So, Feynman rule would be:

- A_μ^A gauge, $\bar{\lambda}_\alpha^B$ gaugino, $\bar{\lambda}_\beta^C$ gaugino in NAC parameter C :

$$\begin{aligned}
& iC^{\mu\nu}q_\nu\epsilon^{\dot{\alpha}\dot{\beta}}d^{ABC}\gamma^{ABC} - \frac{i}{8}XC^{\mu\nu}q_\nu\epsilon^{\dot{\alpha}\dot{\beta}}d^{ABC}\kappa^{BAC} \\
& + \frac{i}{4}Y^{\mu\nu\dot{\alpha}\dot{\beta}}(p_1 - p_2)_\nu d^{ABC}\kappa^{BAC}
\end{aligned} \tag{112}$$

- A_μ^C gauge, A_μ^D gauge, $\bar{\lambda}_\alpha^A$ gaugino, $\bar{\lambda}_\beta^B$ gaugino in NAC parameter C :

$$\begin{aligned}
& \frac{1}{2}gC^{\mu\nu}\epsilon^{\dot{\alpha}\dot{\beta}}d^{ABE}f^{CDE}\gamma^{EAB} - \frac{1}{4}gC^{\mu\nu}\epsilon^{\dot{\alpha}\dot{\beta}}d^{BDE}f^{ACE}\kappa^{EDB} \\
& + \frac{1}{2}Y^{\mu\nu\dot{\alpha}\dot{\beta}}d^{BDE}f^{ACE}\kappa^{EDB}
\end{aligned} \tag{113}$$

E : Counter Terms

We have made use from the follow Counter Terms in the procedure of the renormalization:

$$\begin{aligned}
\Gamma_{C.T}^{\mu\dot{\alpha}\dot{\beta}} = & +id^{abc}(\delta_{01}\gamma^{abc} - \frac{\delta_1}{8}\kappa_1)\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\
& +id^{000}(\delta_{02}\gamma^{000} - 0)\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\
& +id^{a0c}(\delta_{03}\gamma^{a0c} - 0)\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\
& +id^{ab0}(\delta_{04}\gamma^{ab0} - \frac{\delta_4}{8}\kappa_3)\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\
& +id^{0bc}(\delta_{05}\gamma^{0bc} - 0)\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu}q_\nu \\
& + \frac{i}{4}\delta_{12}d^{abc}\kappa_1(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}(p_1 - p_2)_\nu \\
& + \frac{i}{4}\delta_{13}d^{ab0}\kappa_3(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}(p_1 - p_2)_\nu
\end{aligned} \tag{114}$$

$$\begin{aligned}
\Gamma_{C.T}^{\mu\nu\dot{\alpha}\dot{\beta}} = & \frac{1}{2}(\delta_{06}\gamma^{eab} - \frac{\delta_6}{4}\kappa_1)d^{abe}f^{cde}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} \\
& + \frac{1}{2}(\delta_{07}\gamma^{e0b} - \frac{\delta_7}{2}\kappa_3)d^{0be}f^{cde}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} \\
& + \frac{1}{2}(\delta_{08}\gamma^{ea0} - 0)d^{a0e}f^{cde}\epsilon^{\dot{\alpha}\dot{\beta}}C^{\mu\nu} \\
& + \frac{1}{4}\delta_{14}gf^{abe}d^{cde}\kappa_1(Y^{\mu\nu})^{\dot{\alpha}\dot{\beta}}
\end{aligned} \tag{115}$$

$$\begin{aligned}
\Gamma_{C.T}^{\dot{\alpha}\dot{\beta}\dot{\gamma}} = & \frac{\delta_{09}}{8}i\gamma_0\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}}d^{abe}d^{cde}|C|^2 + \frac{\delta_{010}}{4N}i\gamma_1\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}}|C|^2 \\
& + \frac{\delta_{011}}{N}i\gamma_2\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}}|C|^2
\end{aligned} \tag{116}$$

F : U(N) Group Identities

$$\begin{aligned}
f^{ABC} &= -f^{ACB} = \dots, \quad d^{ABC} = d^{ACB} = \dots \\
f^{CAD} f^{DBC} &= -N c^A \delta^{AB}, \\
d^{CAD} d^{DBC} &= N d^A \delta^{AB}, \\
f^{CAD} d^{DBC} &= 0, \\
f^{DAE} f^{EBF} f^{FCD} &= -\frac{N}{2} f^{ABC}, \\
d^{DAE} f^{EBF} f^{FCD} &= -\frac{N}{2} d^{ABC} d^A c^B c^C, \\
f^{DAE} d^{EBF} d^{FCD} &= \frac{N}{2} f^{ABC}, \\
d^{IAJ} f^{JBK} f^{KCL} f^{LDI} &= -\frac{N}{4} [d^{ABE} f^{CDE} + f^{ABE} d^{CDE}] d^A c^B c^C c^D, \\
d^{ABCD} &= \text{Tr}[F^A F^B D^C D^D] = c^A c^B [\frac{1}{2} c^C c^D (\delta_{AC} \delta_{BD} - \delta_{AB} \delta_{CD} + \delta_{AD} \delta_{BC}) \\
&\quad + \frac{N}{8} d^C d^D (-f^{ABE} f^{CDE} - f^{ACE} f^{BDE} - d^{ABE} d^{CDE} - d^{ACE} d^{BDE})], \\
\tilde{d}^{abcd} &= \text{Tr}[F^a D^c F^b D^d], \tag{117}
\end{aligned}$$

Where $d^A = 1 + \delta^{0A}$, $c^A = 1 - \delta^{0A}$. The gauge index of U(N) runs $A = 0, 1, \dots, N^2 - 1$ where $A = 0$ corresponds to overall U(1) while $A = a = 1, \dots, N^2 - 1$ corresponds to SU(N). For the simplicity of the calculation, we introduce the matrix F^A and D^A , whose component is given by f^{ABC} and d^{ABC} as

$$(F^A)_{BC} = f^{BAC}, \quad (D^A)_{BC} = d^{BAC}$$

Taking into account that f^{ABC} is totally antisymmetric tensor and d^{ABC} is totally symmetric tensor.

G :Sigma Matrices

$$\sigma_{\alpha\dot{\alpha}}^\mu \bar{\sigma}_{\dot{\mu}}^{\beta\beta} = -2\delta_\alpha^\beta \delta_{\dot{\alpha}}^{\dot{\beta}}$$

$$\begin{aligned}
\sigma_{\alpha\dot{\alpha}}^{\mu}\sigma_{\mu\beta\dot{\beta}} &= -2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}} \\
\bar{\sigma}^{\mu\dot{\alpha}\alpha}\bar{\sigma}_{\mu}^{\dot{\beta}\beta} &= -2\epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}} \\
(\sigma^{\mu}\bar{\sigma}_{\mu})_{\alpha}^{\beta} &= -4\delta_{\alpha}^{\beta} \\
(\bar{\sigma}^{\mu}\sigma_{\mu})_{\dot{\beta}}^{\dot{\alpha}} &= -4\delta_{\dot{\beta}}^{\dot{\alpha}} \\
(\sigma^{\mu}\bar{\sigma}^{\nu})_{\alpha}^{\beta} &= 2(\sigma^{\mu\nu})_{\alpha}^{\beta} - g^{\mu\nu}\delta_{\alpha}^{\beta} \\
(\bar{\sigma}^{\mu}\sigma^{\nu})_{\dot{\beta}}^{\dot{\alpha}} &= 2(\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}} - g^{\mu\nu}\delta_{\dot{\beta}}^{\dot{\alpha}} \\
(\sigma^{\mu\nu})_{\alpha}^{\beta}(\sigma_{\mu\nu})_{\rho}^{\kappa} &= 2\delta_{\alpha}^{\kappa}\delta_{\rho}^{\beta} - \delta_{\alpha}^{\beta}\delta_{\rho}^{\kappa} \\
(\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}}(\bar{\sigma}_{\mu\nu})_{\dot{\kappa}}^{\dot{\rho}} &= 2\delta_{\dot{\kappa}}^{\dot{\rho}}\delta_{\dot{\beta}}^{\dot{\alpha}} - \delta_{\dot{\beta}}^{\dot{\alpha}}\delta_{\dot{\kappa}}^{\dot{\rho}} \\
Tr(\sigma^{\mu}\bar{\sigma}^{\nu}) &= Tr(\bar{\sigma}^{\mu}\sigma^{\nu}) = -2g^{\mu\nu} \\
Tr(\sigma^{\mu\nu}) &= Tr(\bar{\sigma}^{\mu\nu}) = 0.
\end{aligned} \tag{118}$$

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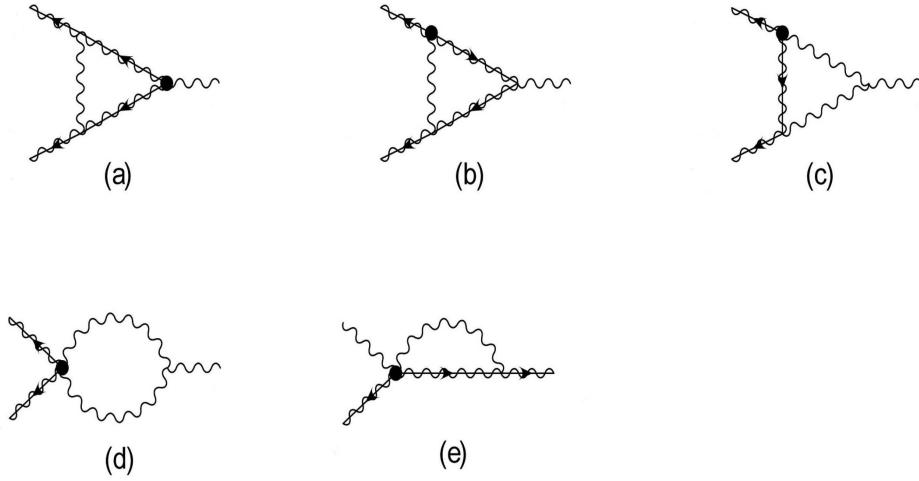


Figure 2: 1PI three point function diagrams with one gauge, two gaugino lines; the black circle represents the positions of NAC parameter C .

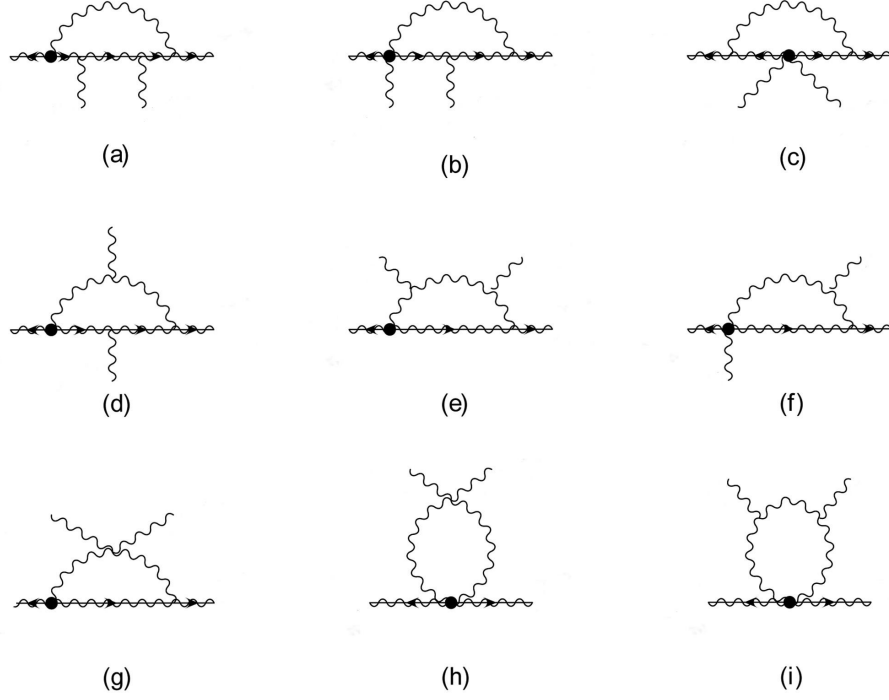


Figure 3: 1PI four point function diagrams with two gauge and two gaugino lines; the black circle represents the positions of a NAC parameter C .

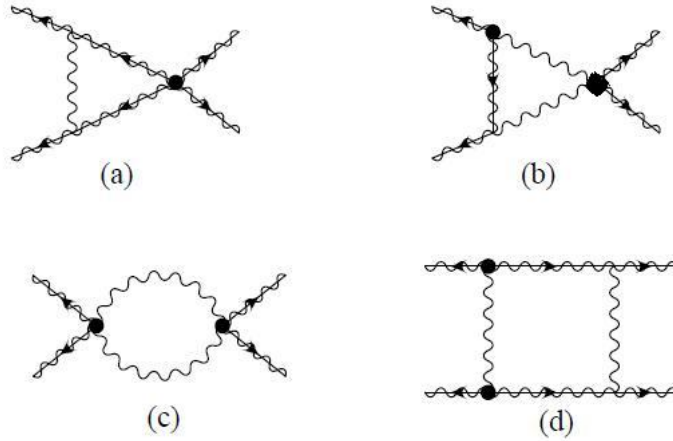


Figure 4: 1PI four point function diagrams with four gaugino lines; the black circle represents the positions of a NAC C .

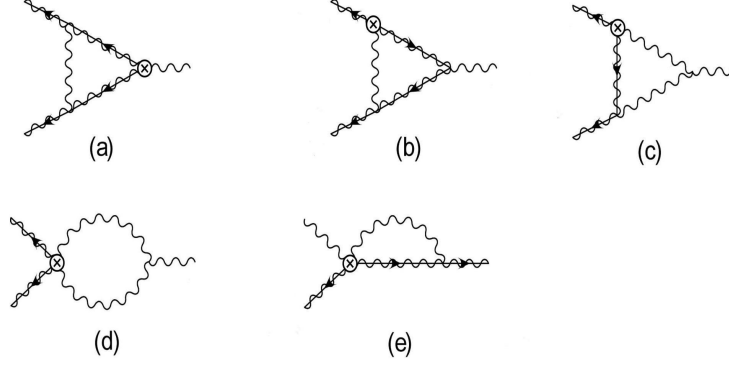


Figure 5: 1PI three point function diagrams with one gauge, two gaugino lines; the crossed circle represents the positions of NAC parameter C associated with parameter Y .

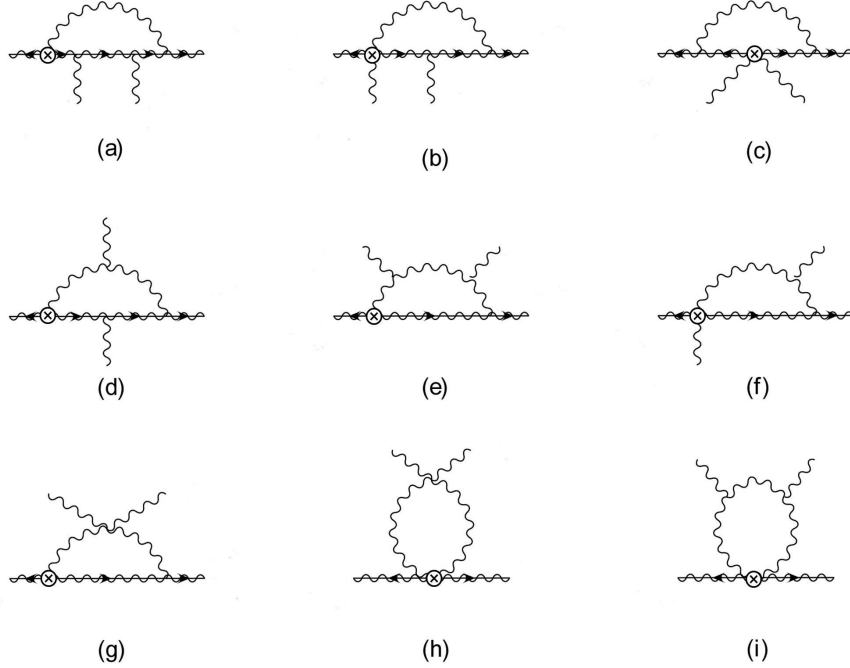


Figure 6: 1PI four point function diagrams with two gauge and two gaugino lines; the crossed circle represents the positions of NAC parameter C associated with parameter Y .